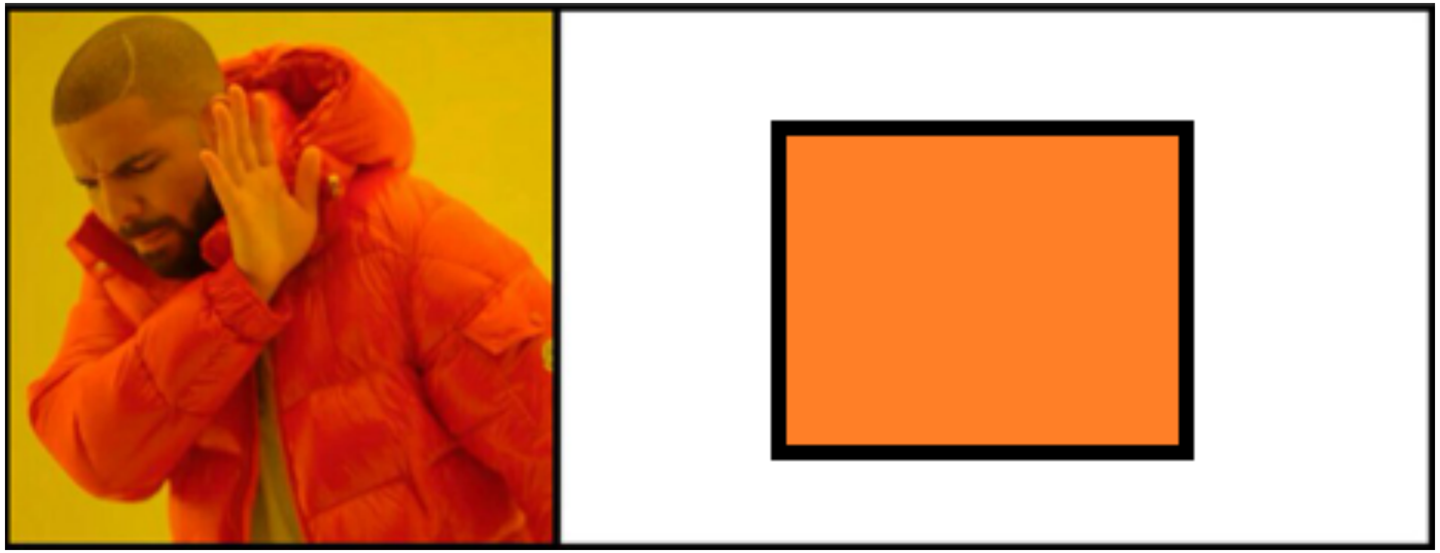
# CONTANDO SECRETOS DE TRES PUNTAS

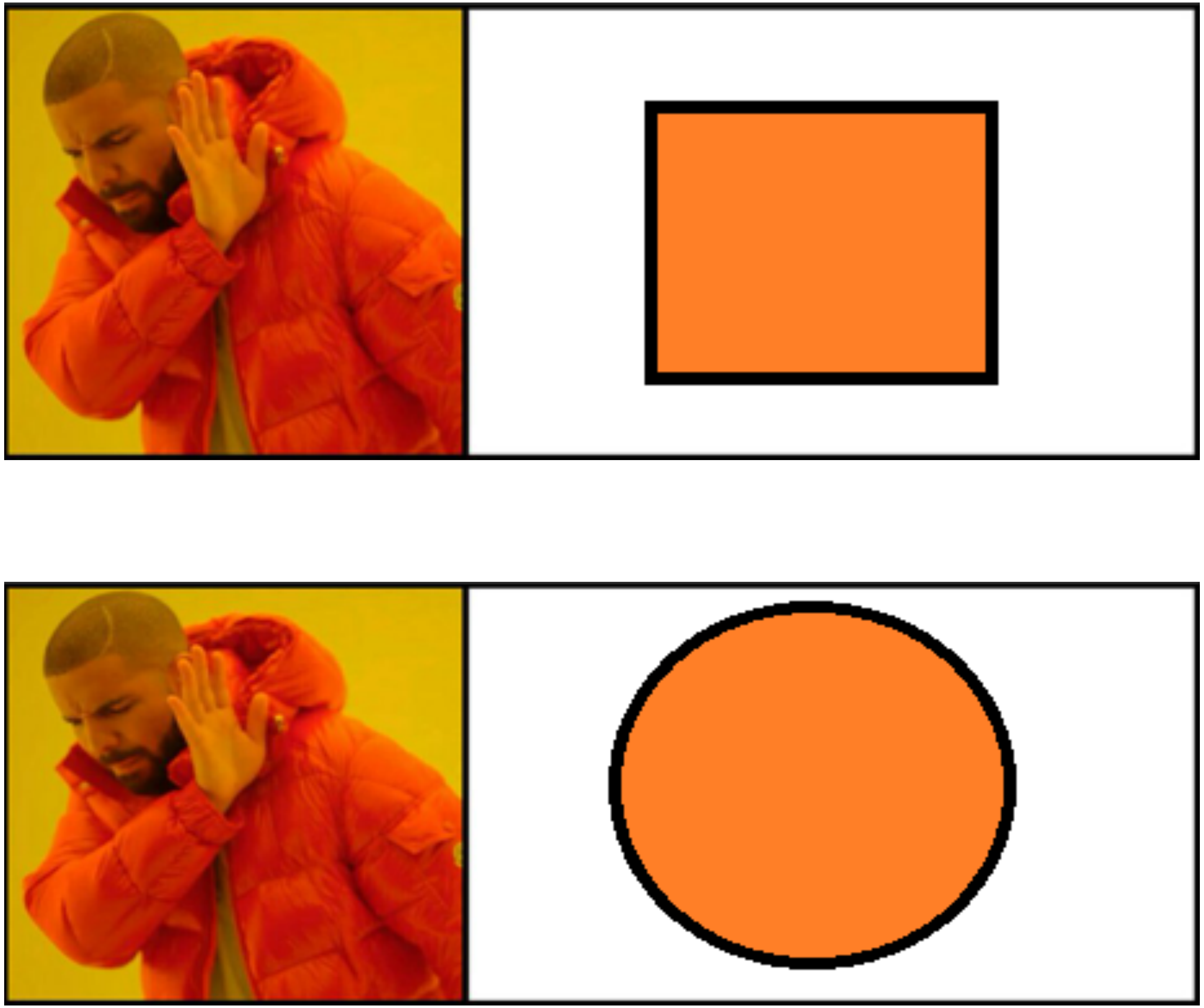
Christian Bartolomé

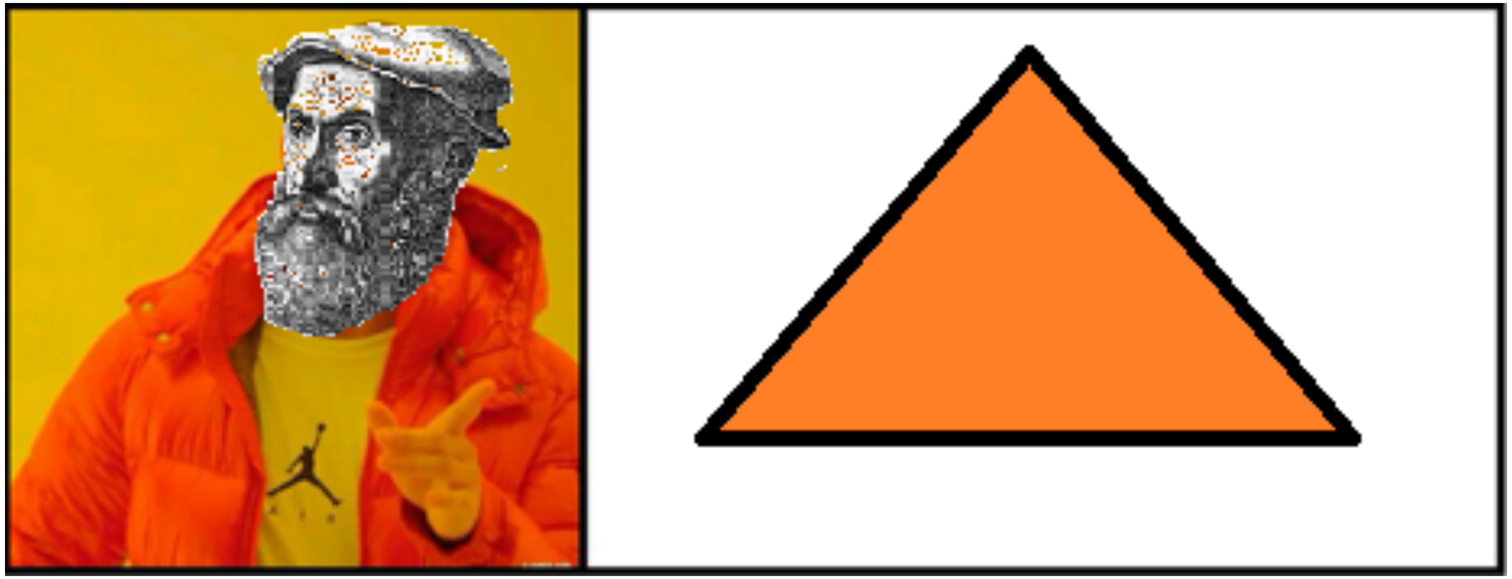
Guillermo Herrera

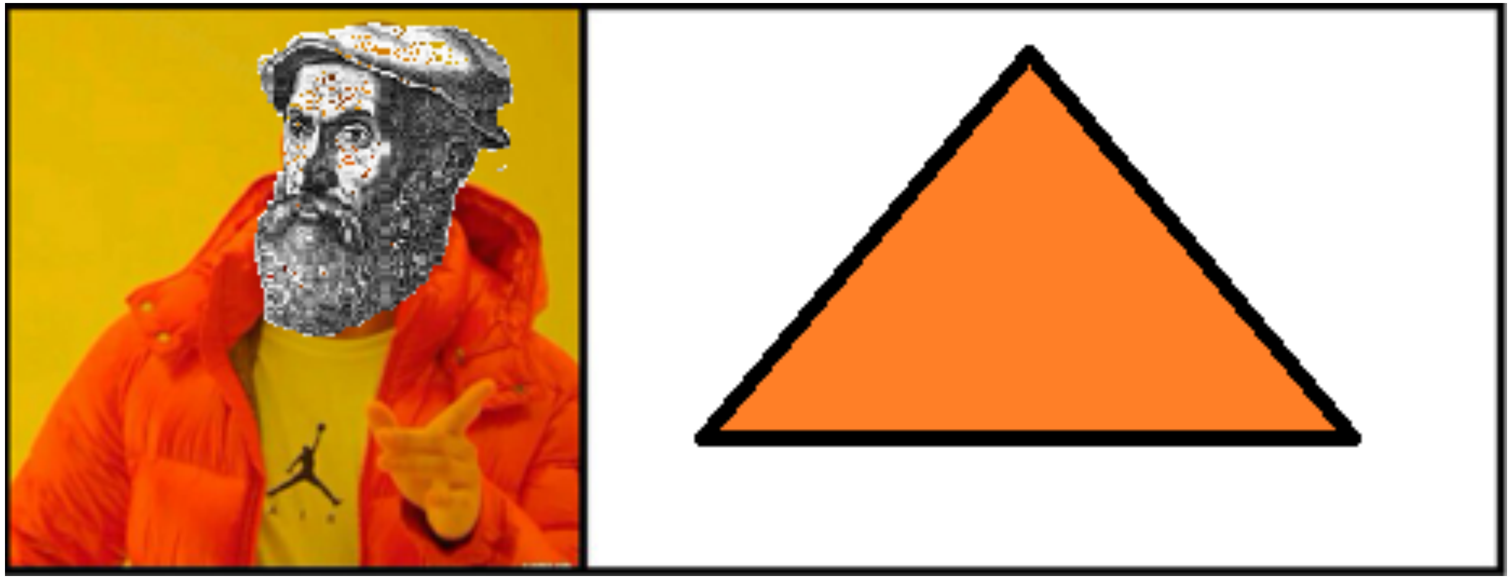
Jueves
28 de febrero de 2019
10:45-10:55
Aula Magna
Matemáticas y Física

POAT-Matemáticas

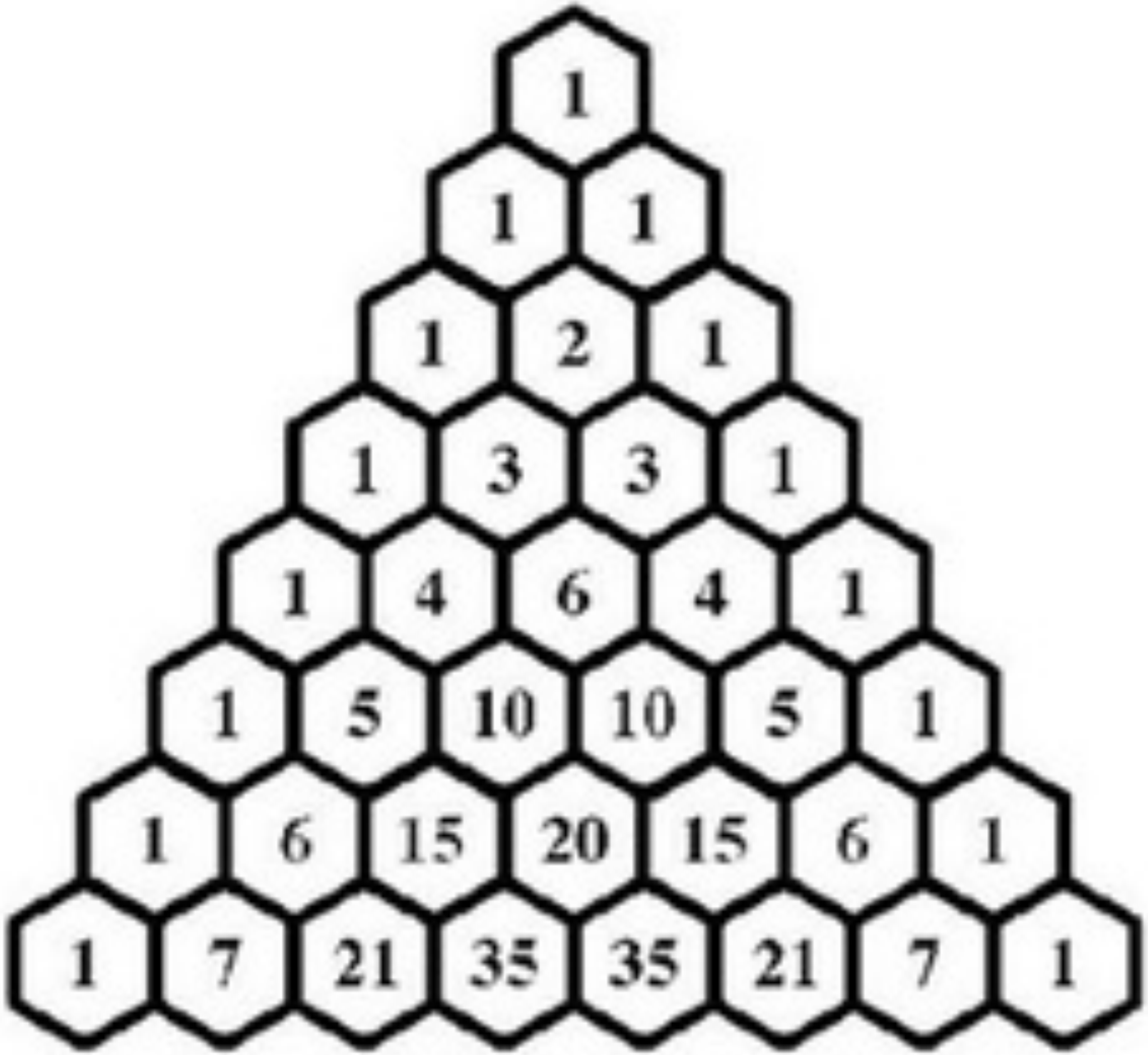








VS


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

# Desarrollo del Binomio $(a+b)^n$

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

# Desarrollo del Binomio $(a+b)^n$

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

$$\longrightarrow \quad (a+b)^0 = \quad 1$$

# Desarrollo del Binomio $(a+b)^n$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$\mathbf{(a+b)^0} = 1$

$\mathbf{(a+b)^1} = 1a + 1b$

# Desarrollo del Binomio $(a+b)^n$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$(a+b)^0 = 1$

$(a+b)^1 = 1a + 1b$

$(a+b)^2 = 1a^2 + 2ab + 1b^2$

# Desarrollo del Binomio $(a+b)^n$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a + 1b$$

$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1ab^3$$

# Desarrollo del Binomio $(a+b)^n$

| | | |
|---|---|---|
| 1 | $(a+b)^0$ | $= 1$ |
| 1 1 | $(a+b)^1$ | $= 1a + 1b$ |
| 1 2 1 | $(a+b)^2$ | $= 1a^2 + 2ab + 1b^2$ |
| 1 3 3 1 | $(a+b)^3$ | $= 1a^3 + 3a^2b + 3ab^2 + 1ab^3$ |
| 1 4 6 4 1 | $(a+b)^4$ | $= 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$ |
| 1 5 10 10 5 1 | $(a+b)^5$ | $= 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$ |

# Potencias de 2

| | |
|---|---|
| 1 | 1 |
| 1 1 | 2 |
| 1 2 1 | 4 |
| 1 3 3 1 | 8 |
| 1 4 6 4 1 | 16 |
| 1 5 10 10 5 1 | 32 |
| 1 6 15 20 15 6 1 | 64 |
| 1 7 21 35 35 21 7 1 | 128 |
| 1 8 28 56 70 56 28 8 1 | 256 |

...

# Los números naturales

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

...

# Los números naturales Y triangulares

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

...

1+2+3+4+5+6+7+…

# Cuadrados Perfectos

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

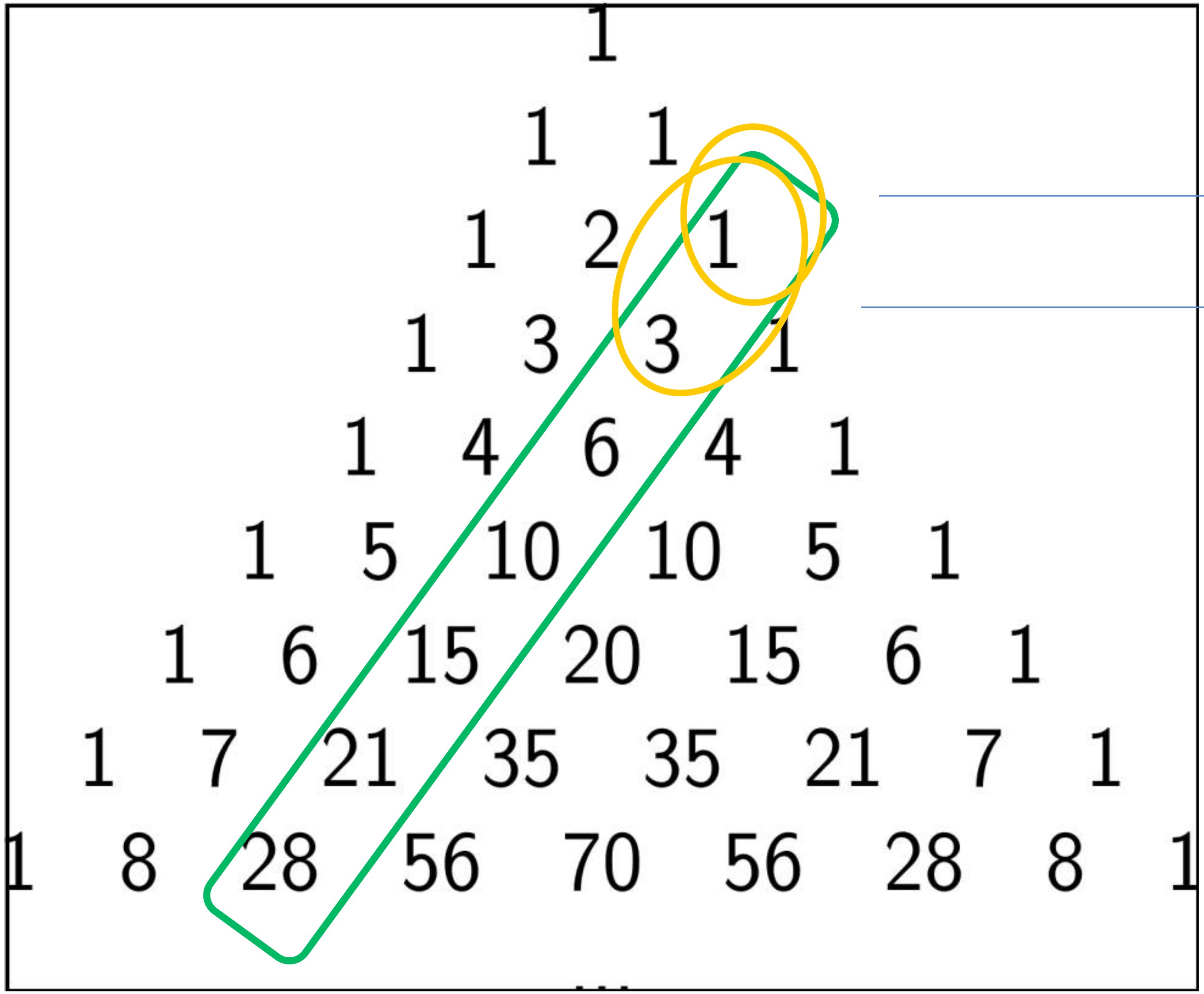
1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

...

1

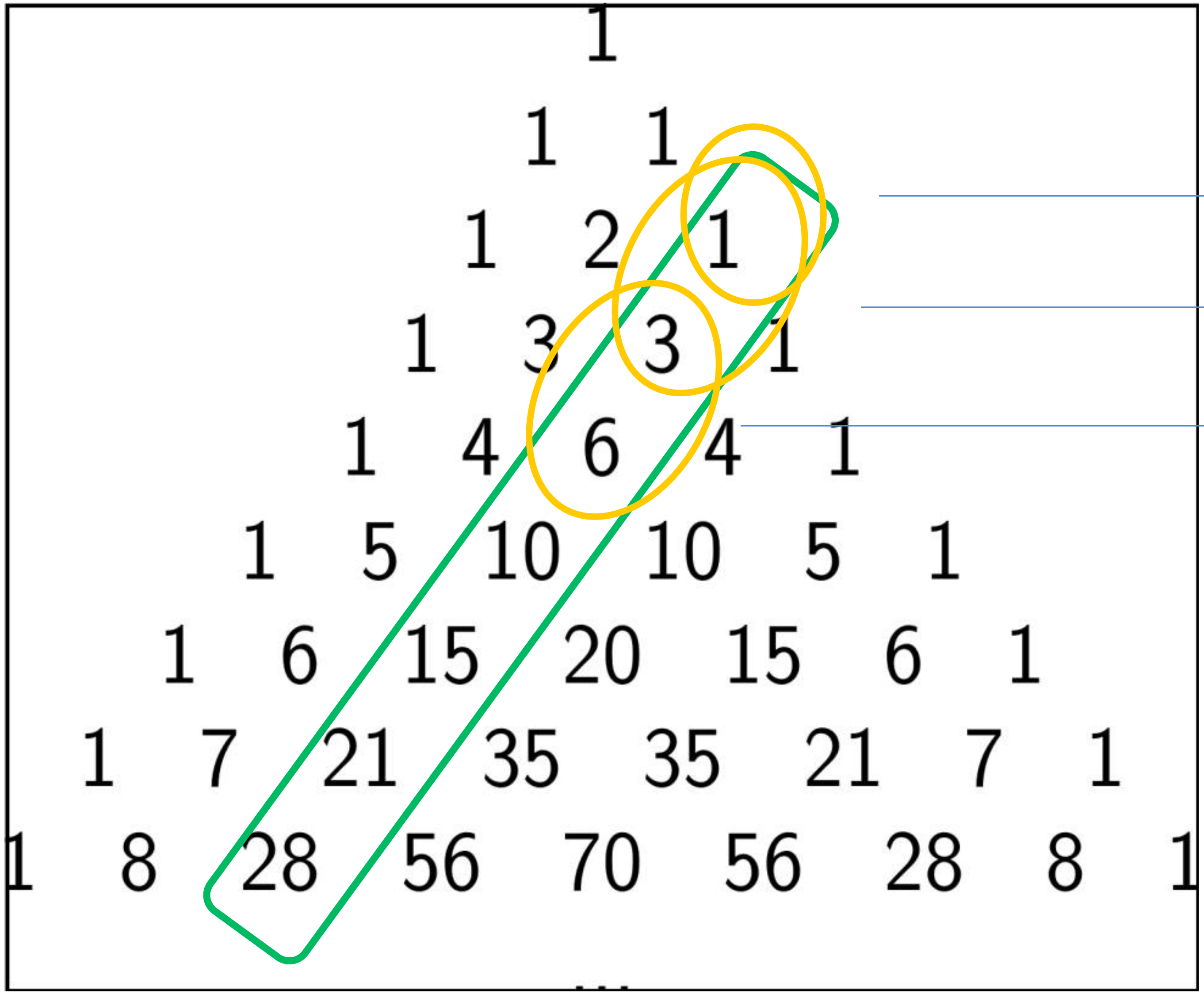
# Cuadrados Perfectos




1

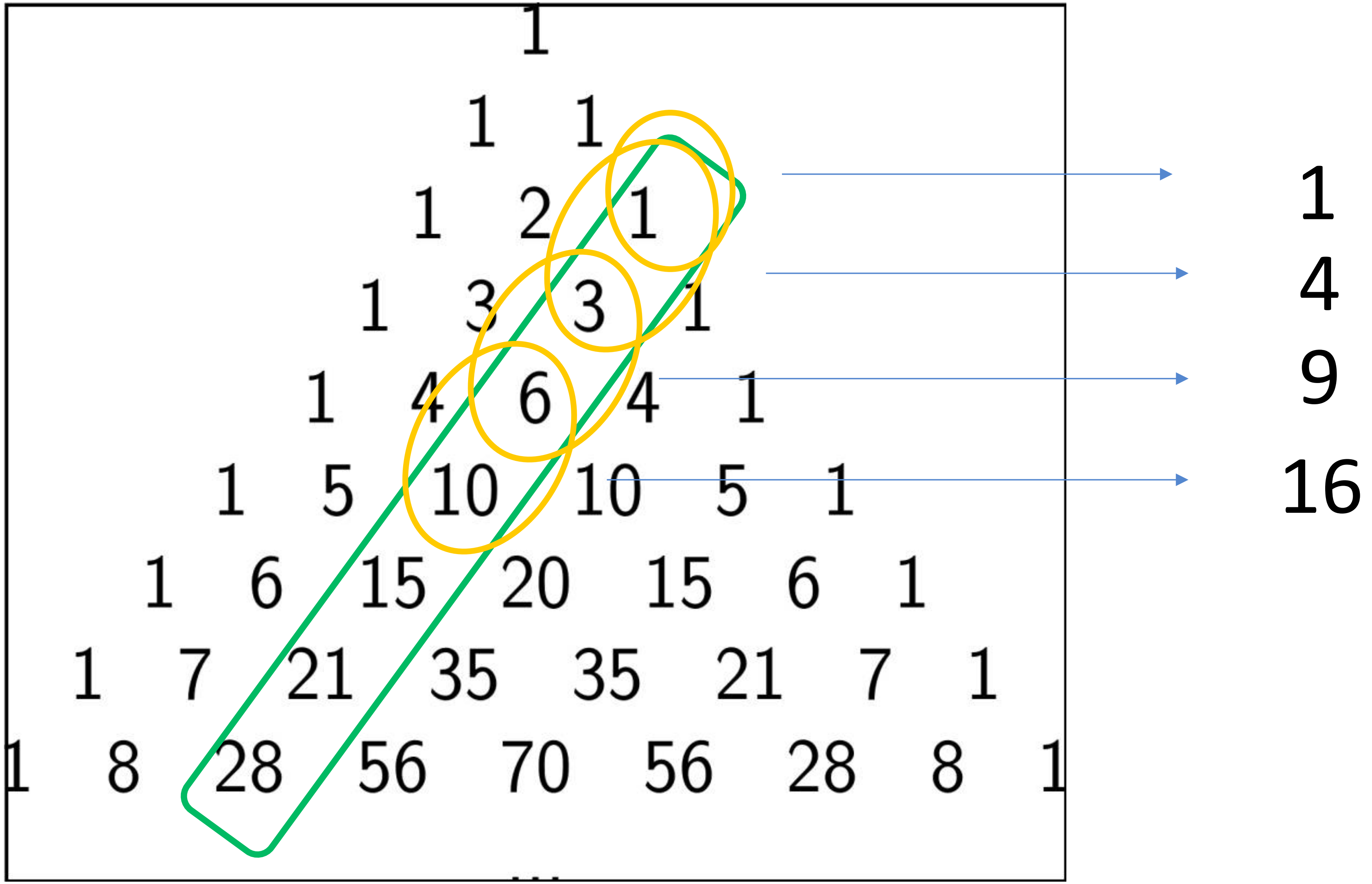
4

# Cuadrados Perfectos

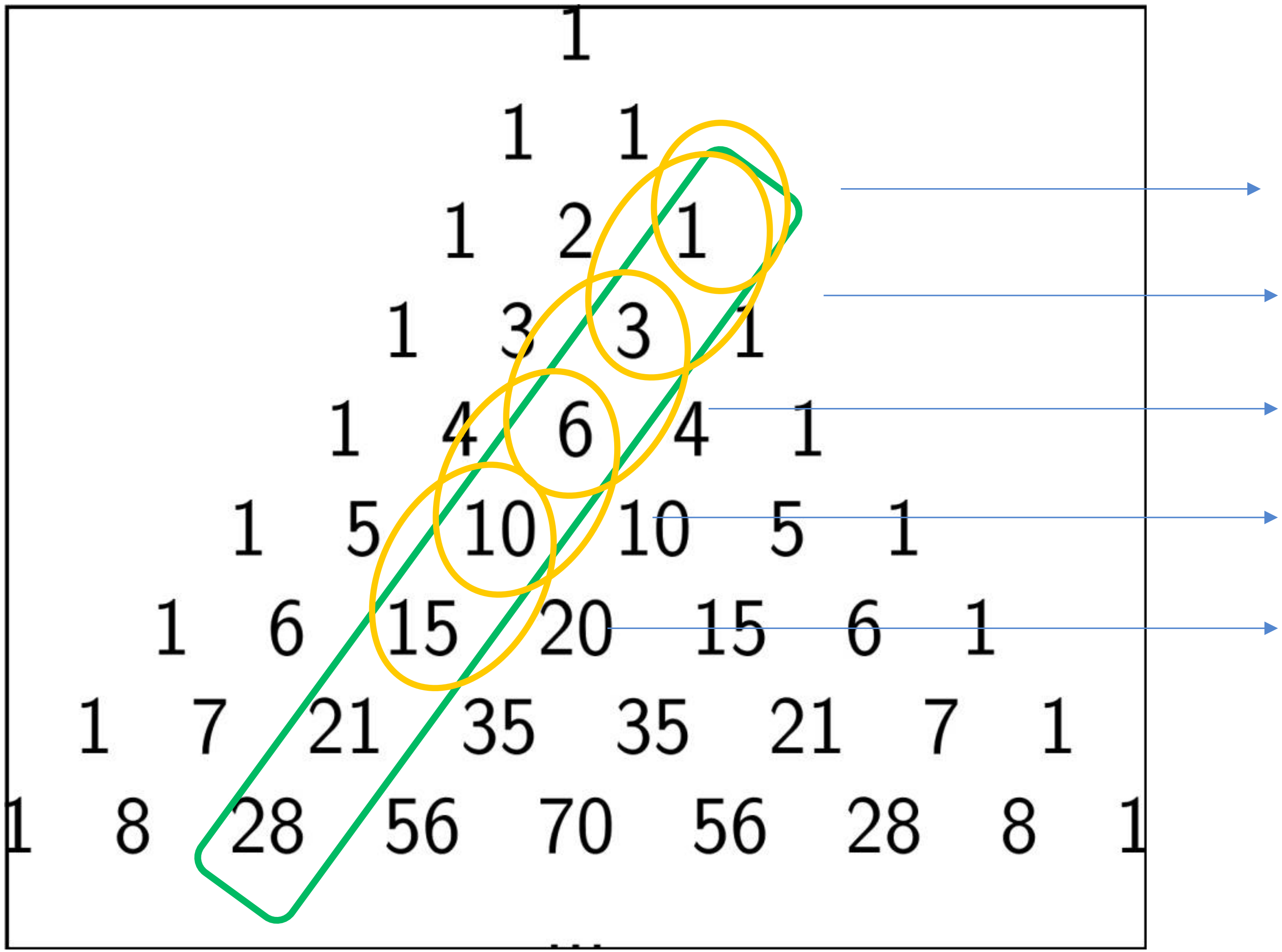



1
4
9

# Cuadrados Perfectos

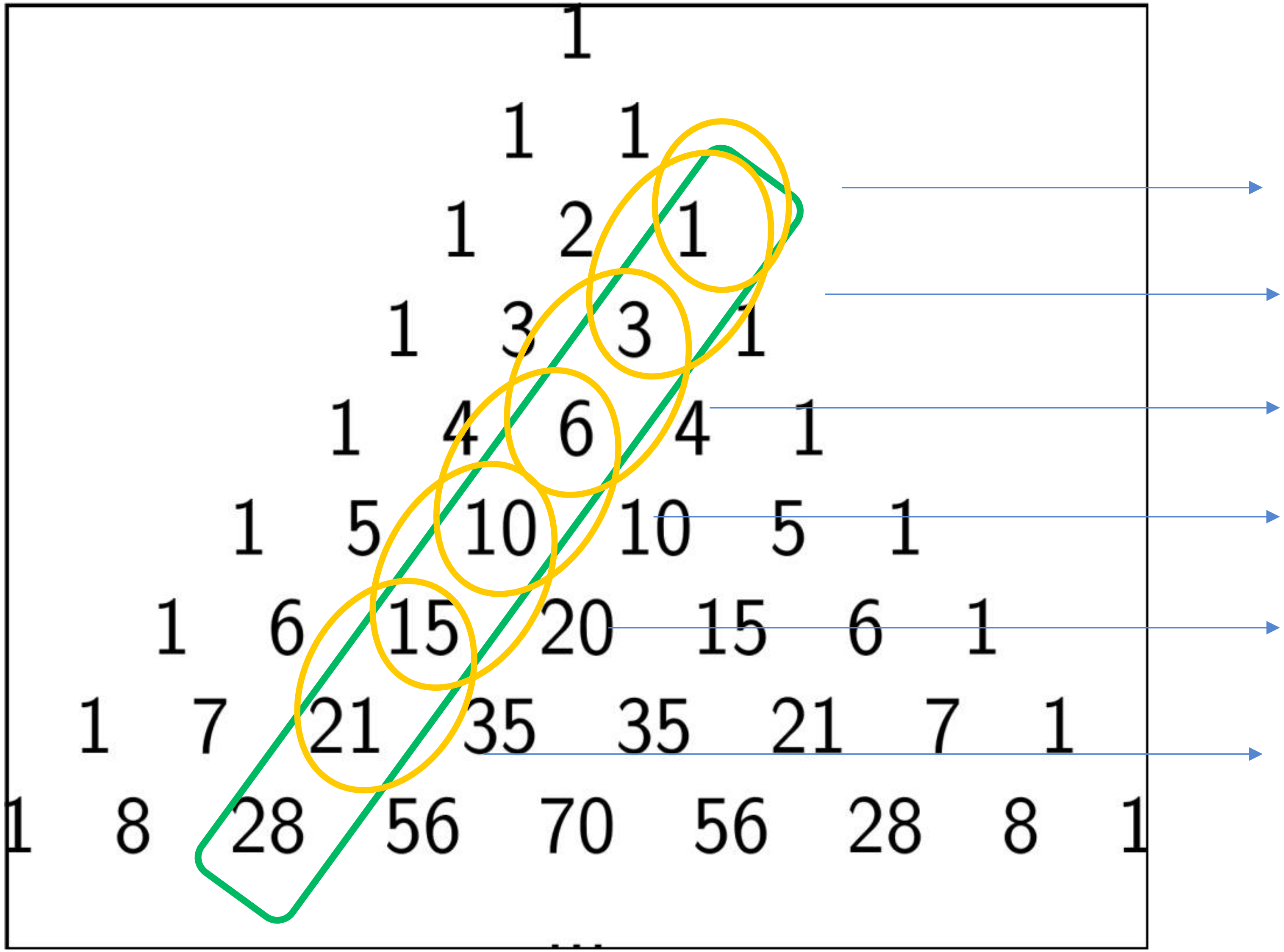


# Cuadrados Perfectos




1
4
9
16
25

# Cuadrados Perfectos




1

4

9

16

25

36

# Filas de múltiplos de números primos

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

...

# Potencias de 11

$1 \longrightarrow 11^0$

$1 \quad 1 \longrightarrow 11$

$1 \quad 2 \quad 1 \longrightarrow 11^2$

$1 \quad 3 \quad 3 \quad 1 \longrightarrow 11^3$

$1 \quad 4 \quad 6 \quad 4 \quad 1 \longrightarrow 11^4$

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

# Potencias de 11

$$
\begin{array}{ccccccccccccl}
 & & & & & 1 & & & & & & \longrightarrow & 11^0 \\
 & & & & 1 & & 1 & & & & & \longrightarrow & 11 \\
 & & & 1 & & 2 & & 1 & & & & \longrightarrow & 11^2 \\
 & & 1 & & 3 & & 3 & & 1 & & & \longrightarrow & 11^3 \\
 & 1 & & 4 & & 6 & & 4 & & 1 & & \longrightarrow & 11^4 \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 & \longrightarrow & 11^5 = 16151
\end{array}
$$

# Potencias de 11

$$1 \longrightarrow 11^0$$

$$1 \quad 1 \longrightarrow 11$$

$$1 \quad 2 \quad 1 \longrightarrow 11^2$$

$$1 \quad 3 \quad 3 \quad 1 \longrightarrow 11^3$$

$$1 \quad 4 \quad 6 \quad 4 \quad 1 \longrightarrow 11^4$$

$$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \longrightarrow 11^5 = 16151$$

1 (5+1) (0+1) (0+5) 1

# Sticks

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

...

# Sticks

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

...

# Sticks

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

...

# Sticks

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

...

# Saliendo de Pesca

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

# Saliendo de Pesca

1

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

# Saliendo de Pesca

1 1

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

# Saliendo de Pesca

1 1 2

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

# Saliendo de Pesca

1 1 2 3

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

# Saliendo de Pesca

1 1 2 3 5

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

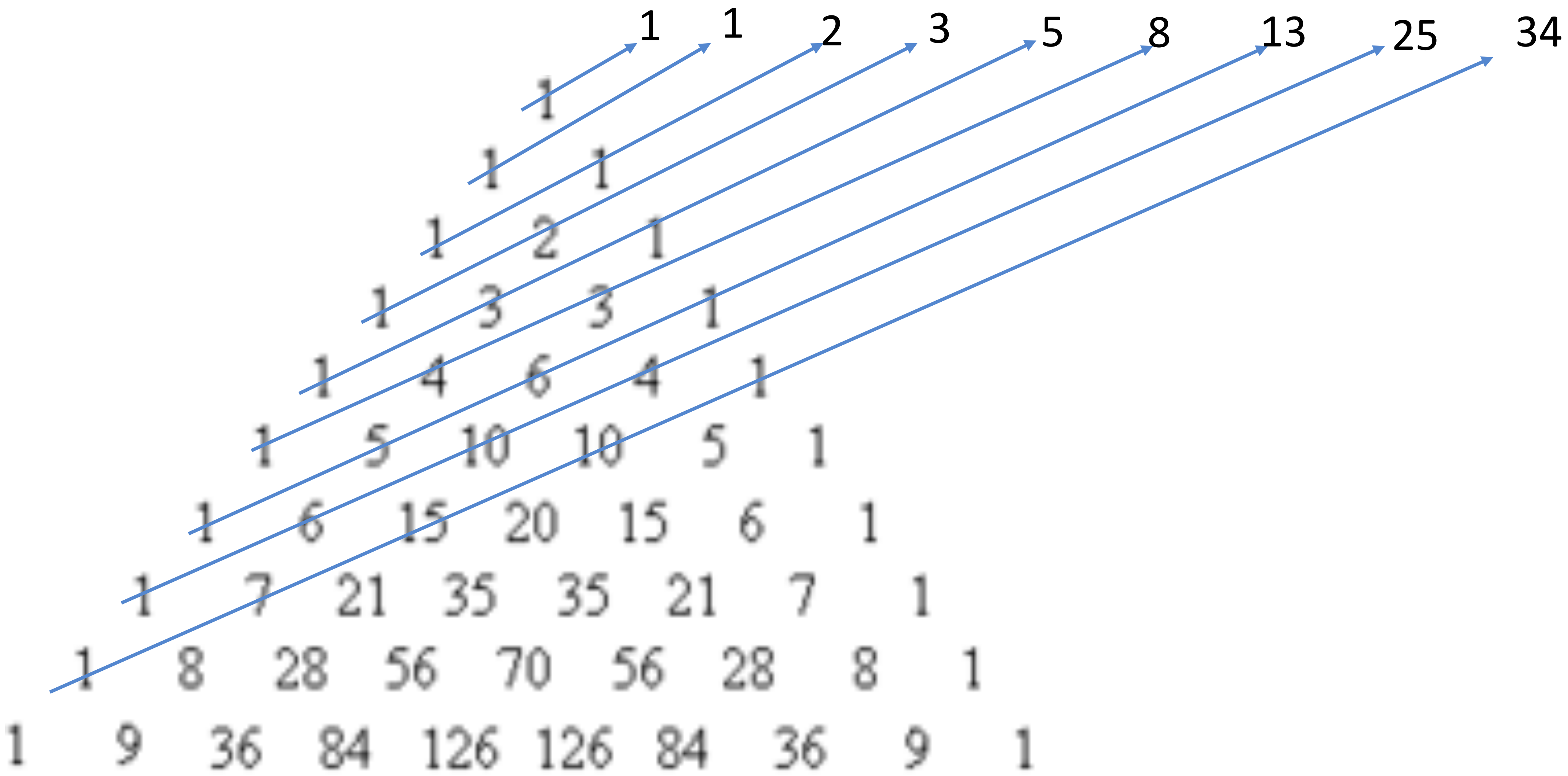
1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

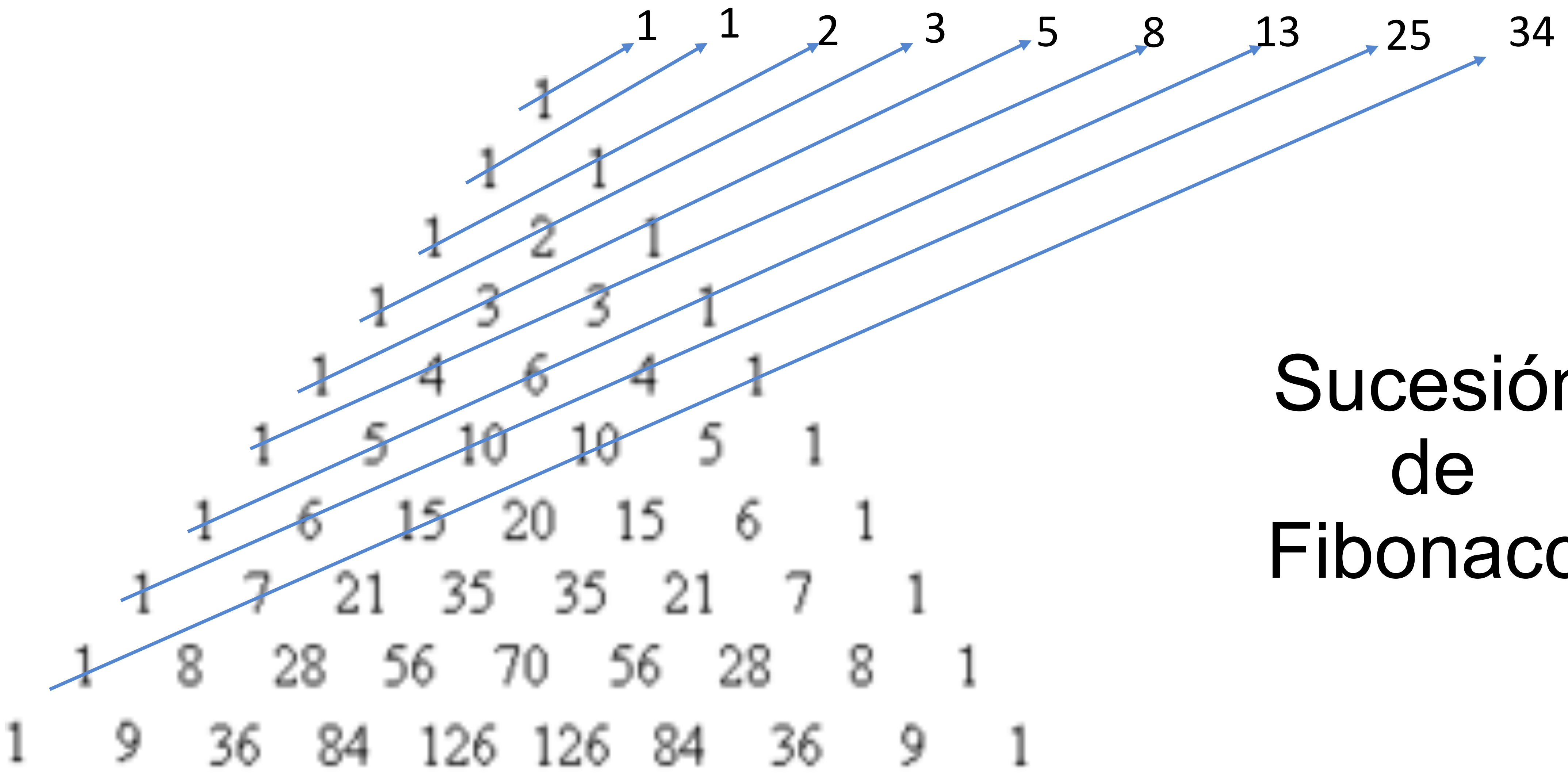
1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

# Saliendo de Pesca

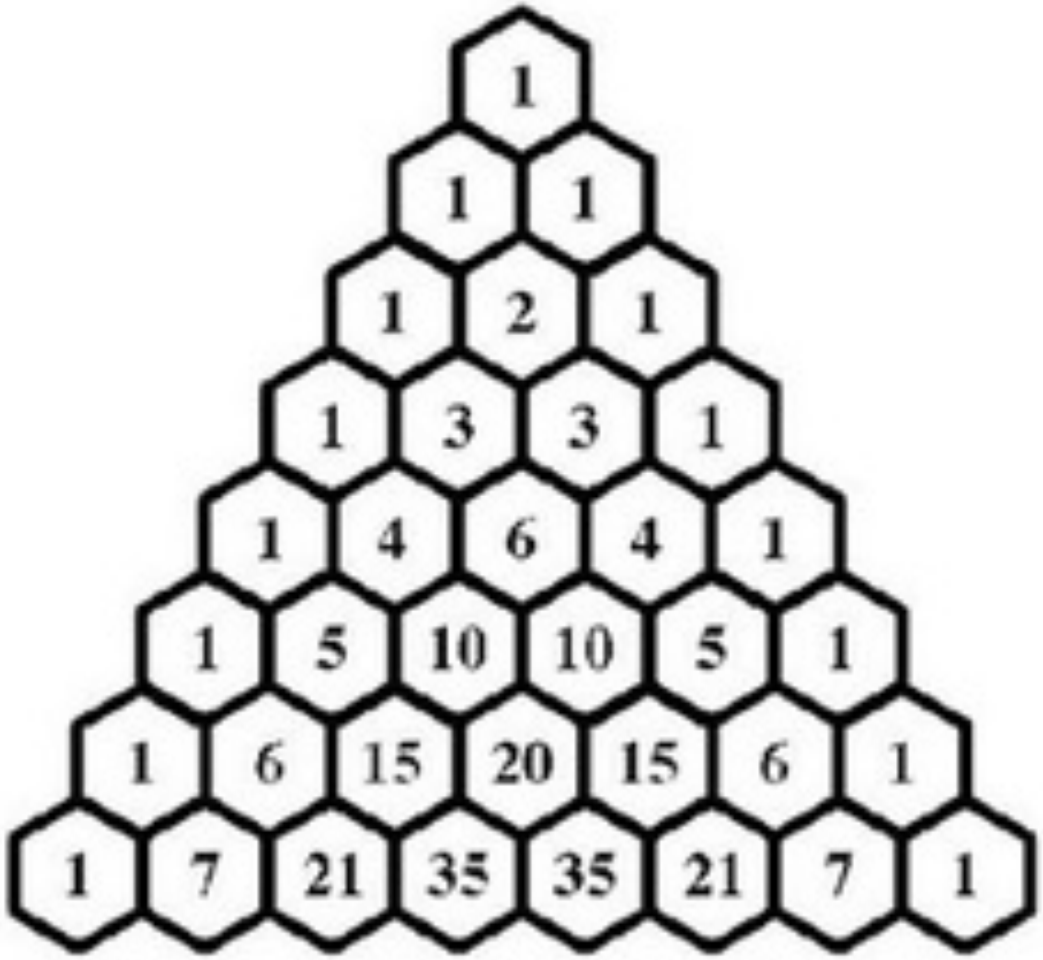


# Saliendo de Pesca

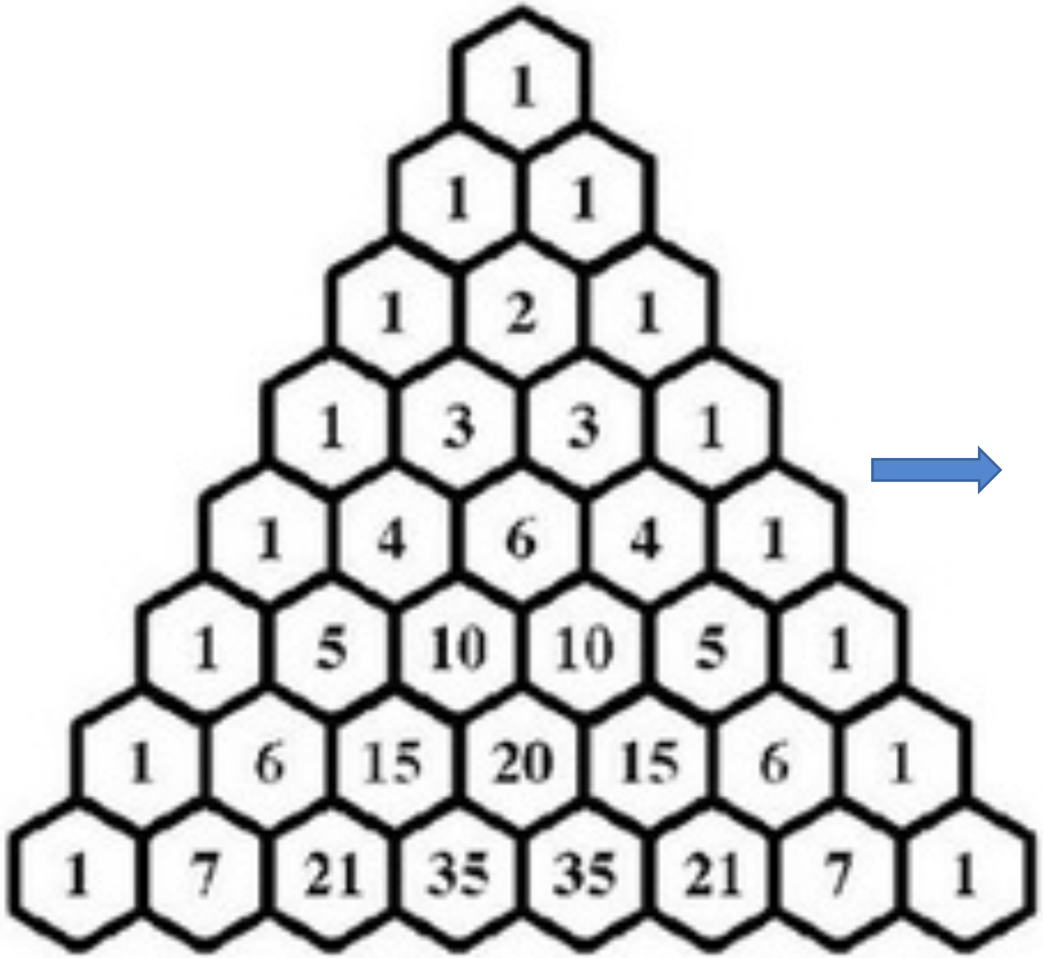



Sucesión
de
Fibonacci

# Primer Gran Ejemplar



# Primer Gran Ejemplar




1

1

2

9

96

2500

162000

26471025

# Primer Gran Ejemplar

{1, 1, 2, 9, 96, 2500, 162000, 26471025, …}

# Primer Gran Ejemplar

{1, 1, 2, 9, 96, 2500, 162000, 26471025, …}

Dividiendo cada término entre el anterior:

{1, 2, 4’5, 10’666…, 26’041666… ,64’8, 163’4013888…}

# Primer Gran Ejemplar

{1, 1, 2, 9, 96, 2500, 162000, 26471025, …}

Dividiendo cada término entre el anterior:

{1, 2, 4’5, 10’666…, 26’041666… ,64’8, 163’4013888…}

De nuevo, dividimos:

{2, 2’25, 2’370370…, 2’44140625…, 2’48832…, 2’52163…}

# Primer Gran Ejemplar

{1, 1, 2, 9, 96, 2500, 162000, 26471025, …}

Dividiendo cada término entre el anterior:

{1, 2, 4’5, 10’666…, 26’041666… ,64’8, 163’4013888…}

De nuevo, dividimos:

{2, 2’25, 2’370370…, 2’44140625…, 2’48832…, 2’52163…}

$n \rightarrow \infty$

# Primer Gran Ejemplar

{1, 1, 2, 9, 96, 2500, 162000, 26471025, …}

Dividiendo cada término entre el anterior:

{1, 2, 4’5, 10’666…, 26’041666… ,64’8, 163’4013888…}

De nuevo, dividimos:

{2, 2’25, 2’370370…, 2’44140625…, 2’48832…, 2’52163…}

$n \longrightarrow \infty$

e

# EL BICHARRACO

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

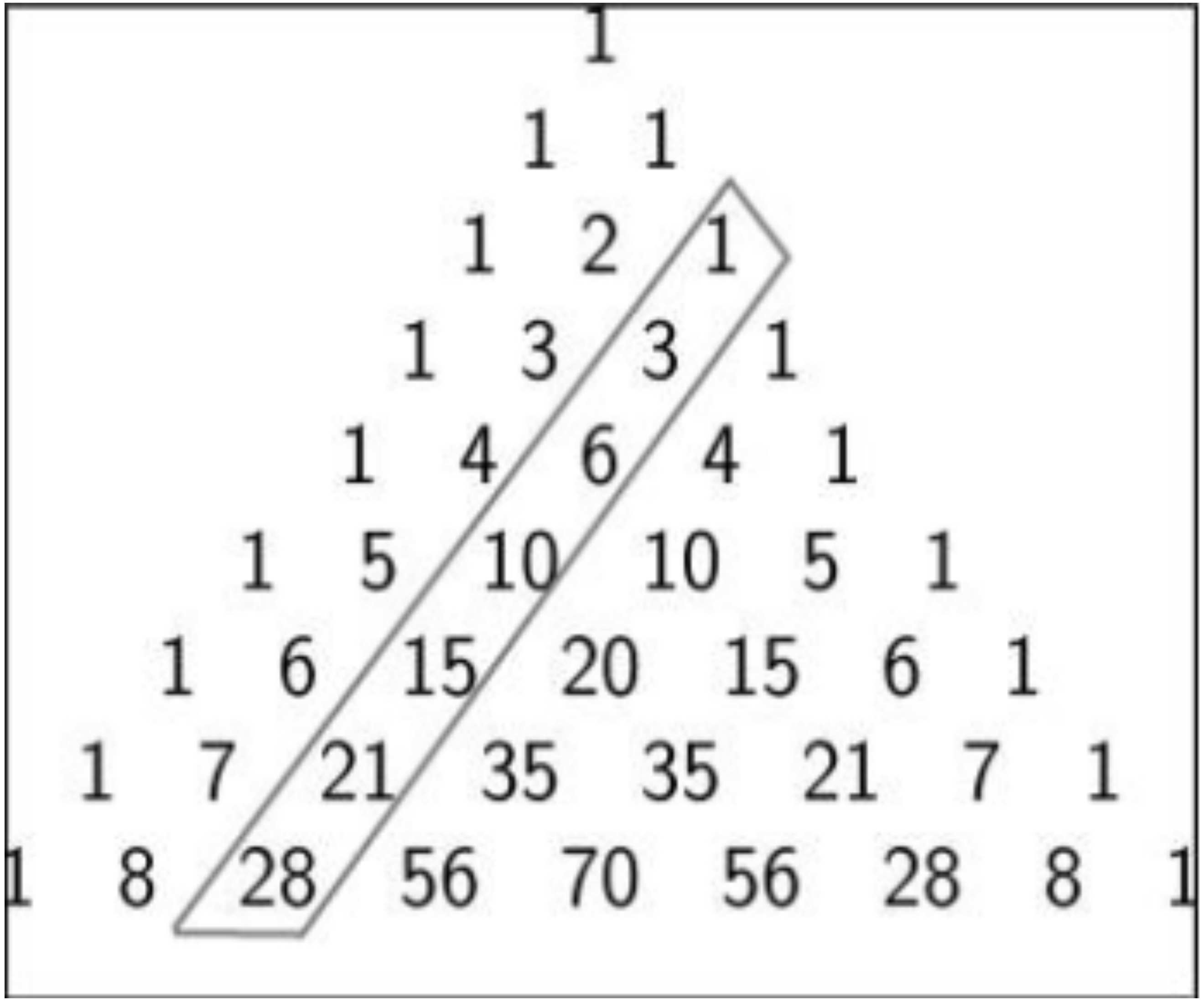
1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

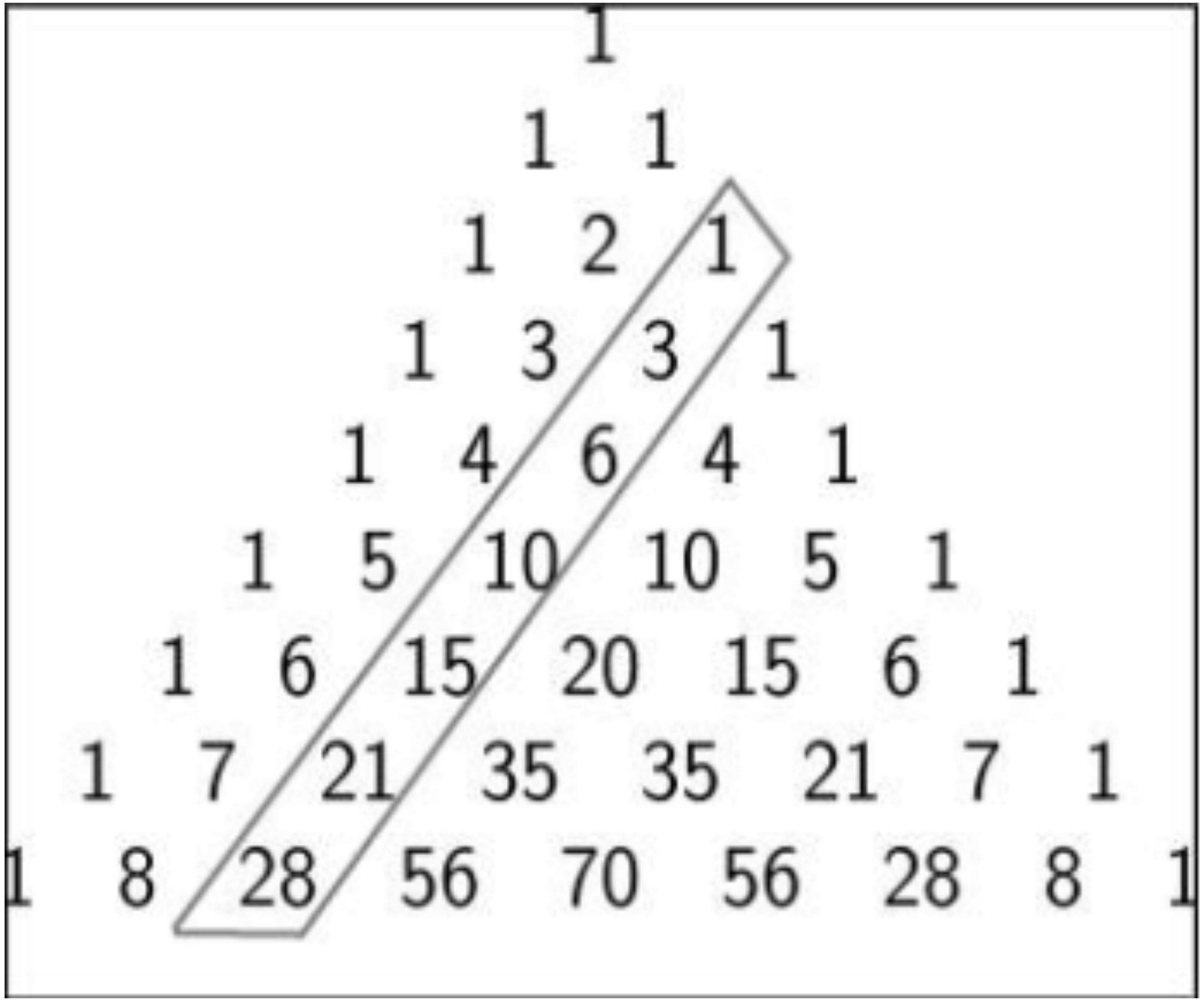
# EL BICHARRACO




Impares sumando, pares restando:

1+3-6-10+15…

# EL BICHARRACO

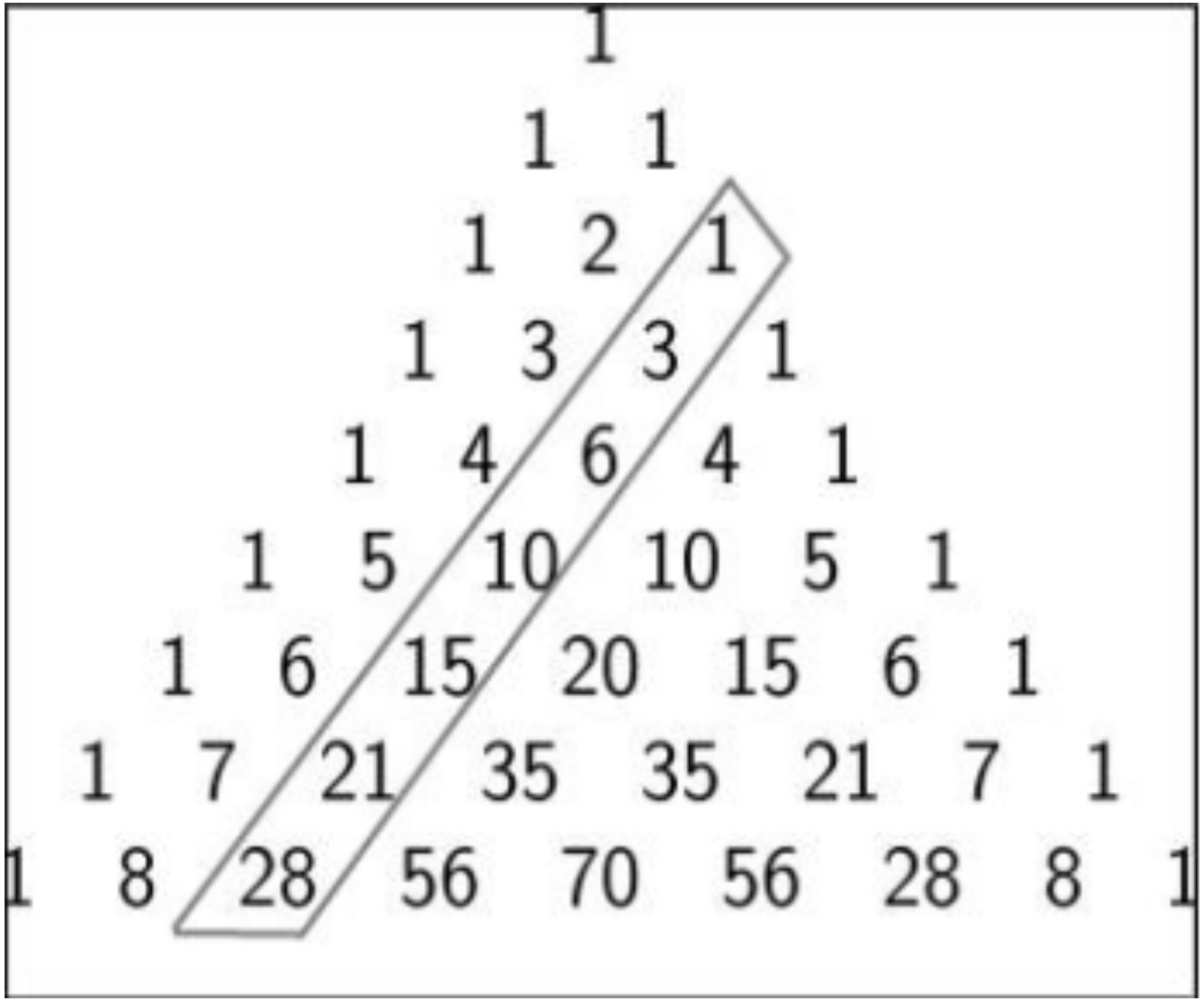



Impares sumando, pares restando:

$$1+3-6-10+15\ldots$$

$$1+ 1/3 - 1/6 - 1/10 + 1/15\ldots$$

# EL BICHARRACO

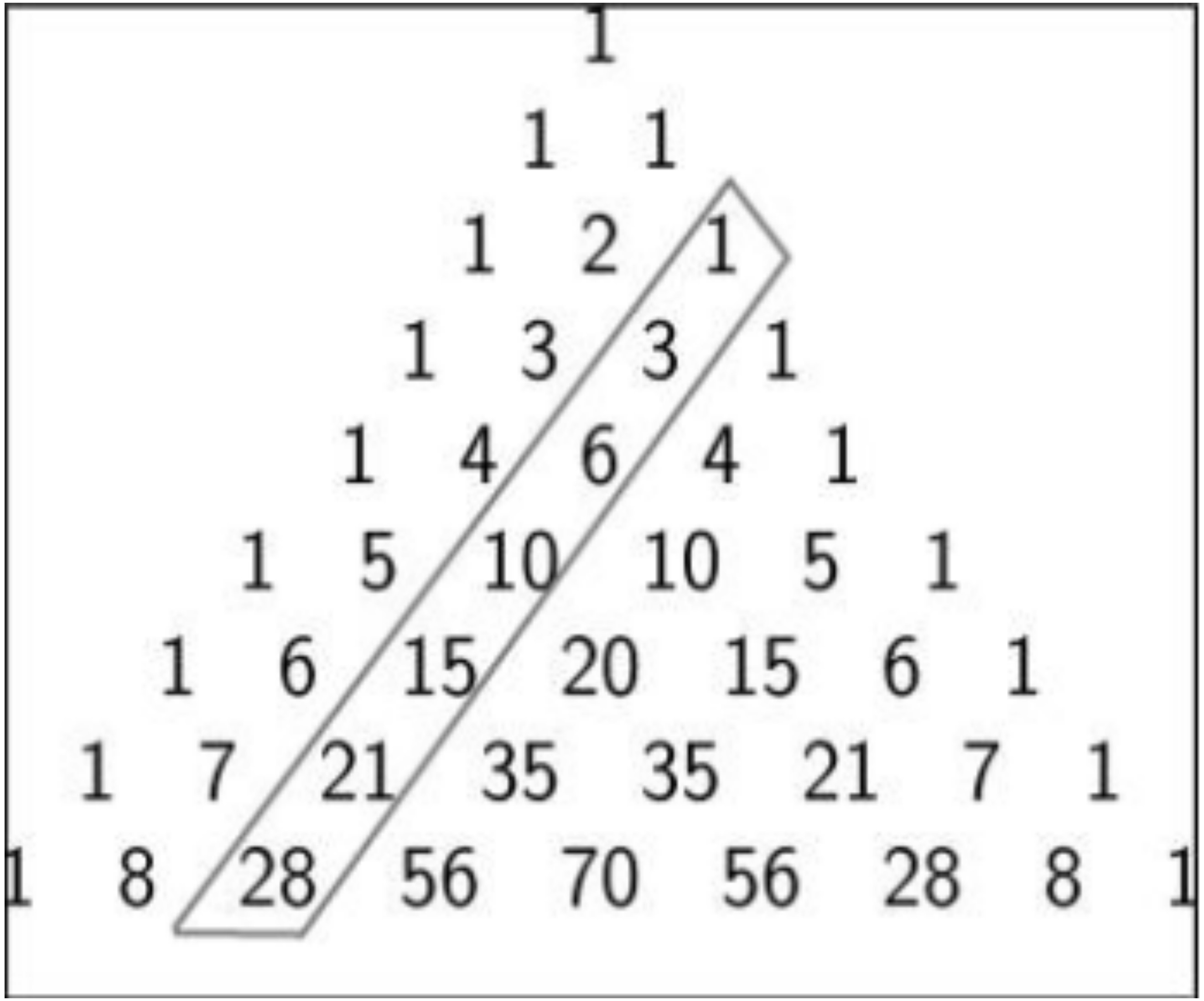



Impares sumando, pares restando:

1+3-6-10+15…

1+ 1/3 - 1/6 - 1/10 + 1/15… +2

# EL BICHARRACO




Impares sumando, pares restando:

1+3-6-10+15…

1+ 1/3 - 1/6 - 1/10 + 1/15… + 2

π

***JONÁS CASTILLO TOLOZA***

***JONÁS CASTILLO TOLOZA***

Matemático que demostró esta convergencia en el año 2007

$$\Pi - 2 = 1+1/3 - 1/6 - 1/10 + 1/15 + 1/21 - 1/28 - 1/36 + \ldots$$

# El Secreto

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 1 | - | - | 1 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 2 | - | - | - | - | 1 | 2 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 3 | - | - | - | - | - | - | 1 | 3 | 3 | 1 | - | - | - | - | - | - | - | - | - | - | - |
| 4 | - | - | - | - | - | - | - | - | 1 | 4 | 6 | 4 | 1 | - | - | - | - | - | - | - | - |
| 5 | - | - | - | - | - | - | - | - | - | - | 1 | 5 | 10 | 10 | 5 | 1 | - | - | - | - | - |
| 6 | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 6 | 15 | 20 | 15 | 6 | 1 | - | - |
| 7 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 7 | 21 | 35 | 35 | 21 | 7 |
| 8 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 8 | 28 | 56 | 70 |

# El Secreto

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 1 | - | - | 1 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 2 | - | - | - | - | 1 | 2 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 3 | - | - | - | - | - | - | 1 | 3 | 3 | 1 | - | - | - | - | - | - | - | - | - | - | - |
| 4 | - | - | - | - | - | - | - | - | 1 | 4 | 6 | 4 | 1 | - | - | - | - | - | - | - | - |
| 5 | - | - | - | - | - | - | - | - | - | - | 1 | 5 | 10 | 10 | 5 | 1 | - | - | - | - | - |
| 6 | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 6 | 15 | 20 | 15 | 6 | 1 | - | - |
| 7 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 7 | 21 | 35 | 35 | 21 | 7 |
| 8 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 8 | 28 | 56 | 70 |

- La **fila n** del triángulo comienza en la **columna 2n**

- Un número de la fila superior es **primo** si cada uno de los elementos de su columna es divisible por su correspondiente número de fila

# El Secreto

| | 0 | 1 | $\boxed{2}$ | $\boxed{3}$ | 4 | $\boxed{5}$ | 6 | $\boxed{7}$ | 8 | 9 | 10 | $\boxed{11}$ | 12 | $\boxed{13}$ | 14 | 15 | 16 | $\boxed{17}$ | 18 | $\boxed{19}$ | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 1 | - | - | 1 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 2 | - | - | - | - | 1 | 2 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 3 | - | - | - | - | - | - | 1 | 3 | 3 | 1 | - | - | - | - | - | - | - | - | - | - | - |
| 4 | - | - | - | - | - | - | - | - | 1 | 4 | 6 | 4 | 1 | - | - | - | - | - | - | - | - |
| 5 | - | - | - | - | - | - | - | - | - | - | 1 | 5 | 10 | 10 | 5 | 1 | - | - | - | - | - |
| 6 | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 6 | 15 | 20 | 15 | 6 | 1 | - | - |
| 7 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 7 | 21 | 35 | 35 | 21 | 7 |
| 8 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 8 | 28 | 56 | 70 |

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# El Secreto

| | 0 | 1 | $\boxed{2}$ | $\boxed{3}$ | 4 | $\boxed{5}$ | 6 | $\boxed{7}$ | 8 | 9 | 10 | $\boxed{11}$ | 12 | $\boxed{13}$ | 14 | 15 | 16 | $\boxed{17}$ | 18 | $\boxed{19}$ | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 1 | - | - | 1 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 2 | - | - | - | - | 1 | 2 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 3 | - | - | - | - | - | - | 1 | 3 | 3 | 1 | - | - | - | - | - | - | - | - | - | - | - |
| 4 | - | - | - | - | - | - | - | - | 1 | 4 | 6 | 4 | 1 | - | - | - | - | - | - | - | - |
| 5 | - | - | - | - | - | - | - | - | - | - | 1 | 5 | 10 | 10 | 5 | 1 | - | - | - | - | - |
| 6 | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 6 | 15 | 20 | 15 | 6 | 1 | - | - |
| 7 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 7 | 21 | 35 | 35 | 21 | 7 |
| 8 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 8 | 28 | 56 | 70 |

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- Un número de la fila superior es **primo** si cada uno de los elementos de su columna es divisible por su correspondiente número de fila

# El Secreto

| | 0 | 1 | $\boxed{2}$ | $\boxed{3}$ | 4 | $\boxed{5}$ | 6 | $\boxed{7}$ | 8 | 9 | 10 | $\boxed{11}$ | 12 | $\boxed{13}$ | 14 | 15 | 16 | $\boxed{17}$ | 18 | $\boxed{19}$ | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 1 | - | - | 1 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 2 | - | - | - | - | 1 | 2 | 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 3 | - | - | - | - | - | - | 1 | 3 | 3 | 1 | - | - | - | - | - | - | - | - | - | - | - |
| 4 | - | - | - | - | - | - | - | - | 1 | 4 | 6 | 4 | 1 | - | - | - | - | - | - | - | - |
| 5 | - | - | - | - | - | - | - | - | - | - | 1 | 5 | 10 | 10 | 5 | 1 | - | - | - | - | - |
| 6 | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 6 | 15 | 20 | 15 | 6 | 1 | - | - |
| 7 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 7 | 21 | 35 | 35 | 21 | 7 |
| 8 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 1 | 8 | 28 | 56 | 70 |

- La **fila n** del triángulo comienza en la **columna 2n**

- Un número de la fila superior es **primo** si cada uno de los elementos de su columna es divisible por su correspondiente número de fila

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

1 12 66 220 495 792 924 792 495 220 66 12 1

1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1

1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1

1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1


¡Muchas
gracias!